


# Migration Models Based on Diffusion and Determinants Gradients: Beyond the Gravity Theory


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## Abstract

*The article proposes an alternative approach to study migration flows based on gravity models. This approach does not reject gravity theory; on the contrary, it expands it to some extent. The relevant models are suggested to be described using diffusion-convection approaches. The intensity of human flows is proposed to identify on the basis of determinant gradients, and the very structure of the domain for studying these processes can be represented as a graph with nodes in the form of continuous areas of social space. The proposed approach is suitable for different dimensions of mobility studying, i.e. permanent migration decisions, touristic flows, academic mobility, transportation etc. The developed conceptual approach and mathematical formalization allow for understanding the patterns of migration applying fundamental principles of mathematic physics for economic processes.*

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## 1. Introduction

Migration and the factors driving people to change their place of residence – especially for long periods – have become some of the most discussed issues in economics, sociology, and other social issues. In Europe, such studies have become particularly relevant since the outbreak of the war in Ukraine. On the one hand, massive flows of forced migrants create budgetary pressure and social tension in host communities (Badalič, 2023; Bilan et al., 2025; Burliai et al., 2023; Palmer & Piper, 2023; Yurchyk et al., 2023; Zayats et al., 2024); on the other hand,

they offer opportunities to gain new resources for economic development and to offset demographic losses (Ashourizadeh et al., 2022; Chugaievska & Wisła, 2023; Kőmüves et al., 2024). The scholarly discourse is gaining increasing relevance against the backdrop of uncertain prospects of military conflicts worldwide and the migration waves they cause. Beyond military factors, large-scale migration driven by other causes (climatic, economic, social) consistently draws attention among the key global risks monitored by the World Economic Forum (WEF, 2022, 2023, 2024, 2025). While the factors that *push* people out of a country are relatively well known and predictable, understanding and modeling migration flows based on *pull* factors of host countries is an interdisciplinary problem. Solving it requires combining the methodological tools of behavioral economics and mathematical modelling.

In social sciences, this problem is often approached from the perspective of neoclassical economic theory, which links migration primarily to employment and income opportunities. A broader range of factors is considered within the *push-pull* theory of migration, whose great advantage is the inclusion of both economic and non-economic determinants. This better reflects current realities and the opportunities for remote work from any country, provided the social environment is favorable.

A common feature of the approaches mentioned is that they all focus on accounting for certain dominant factors influencing the choice of a country to move to. At the same time, their practical application is limited to analysis and modeling based on actual migration flow data and the factors that may affect them. Combining socioeconomic foundations of research with approaches from mathematical physics allow the task of modeling migration flows to be addressed more systematically – by determining the intensity of migration flows to host countries based on a potential function of migration determinants in the social space. This is the *aim* of the present study. In this way, migration is proposed to be considered as a socioeconomic process whose features can be described by the general logic of equations from mathematical physics. This approach is new and, while grounded in gravity theory, is adapted for modeling human resource flows using a *diffusion-convection* type model. The practical application of such an approach in forecasting population migration dynamics is important for developing demographic strategies, managing the socioeconomic sphere, and shaping migration restrictions (or, conversely, incentives) depending on the goals of migration policies.

## 2. Theoretical Framework & Methodology

Research on issues of human *flows* in the social environment has already formed into a distinct scientific field (Pappalardo et al., 2023). Some scholars (Pu et al., 2023; Mishchuk et al., 2024) classify human migration as a matter of strategic public policy because it affects the development of countries and cities. Therefore, it is important to understand the essence of human migration, the methods of its mathematical description and forecasting, in order to develop ways to manage these processes (at the level of cybernetic systems, which human society represents). The importance of studying human mobility in relation to disease epidemics is confirmed by the findings of (Wardle et al., 2023; Yin et al., 2023; Guardabascio et al. 2024; Soriano-Paños et al., 2022). Indeed, understanding, forecasting, and quantitatively assessing migration flows makes it possible to make managerial decisions and influence epidemiological situations.

The authors of the article (Pu et al., 2023) present an in-depth analysis of the significance, characteristics, and development of methods in human migration studies (their review covers

113 primary sources related to human migration and corresponding methods). The problem of translating migration into the language of mathematics is proposed to be solved by adopting as key measurable quantities the migration flows, which reflect the dynamics of the process, and the actual number of migrants. The tasks of studying human migration are divided into two broad categories: the task of forecasting human migration (forecasting HM) and the task of predicting the future development of human migration (predicting future HM development). The second task is precisely the goal of mathematical modeling, aimed at managing these processes.

Among the drivers of human migration, the authors (Pu et al., 2023) distinguish: economic determinants; political determinants; environmental determinants (e.g., the issue of climate change and migration is considered by (Beyer et al., 2023; Lyeonov et al., 2025); social determinants (e.g., as one of the factors influencing migration decision, the diaspora factor is examined by (Prieto-Curiel et al., 2024); demographic determinants. Among the methods for modeling these processes, the authors distinguish: deterministic methods; stochastic methods (which include the gravity model); and Machine Learning Methods.

Within the framework of neoclassical economic theory, the most important determinant of migration is considered to be employment and the income derived from it (Kochaniak et al., 2024; Berde & Remsei, 2025; Virak & Bilan, 2022). One of the well-known models that develops these studies is the Harris-Todaro model, which has been further developed by subsequent researchers and tested in various labor markets. Today it is used not only to explain rural-urban migration flows but also to identify the influence of employment and income differences across labor markets in different countries (Sancar & Akbaş, 2022; Seidu et al., 2022; Tomoiagă & Silaghi, 2023). A larger set of determinants is considered by researchers using the *pull-push* theory of migration, which, in addition to differences in earnings and economic well-being, also emphasizes the impact of factors such as the quality and fairness of social relations, the development of social infrastructure, and the observance of human rights (Aliyev & Gasimov, 2023; Mishchuk & Grishnova, 2015; Ngwu et al., 2023; Perelli-Harris et al., 2023.; Yurchyk et al., 2023; Zavisca et al., 2023).

A detailed review of modeling human migration flows is provided by Barbosa et al. (2018). Among the models of migration flows, as a subtype of *human mobility*, the gravity model is described there, as well as in another work with a fairly comprehensive review section (Soriano-Paños et al., 2022). The study by Capoani (2023) presented a detailed historical review and analysis of the theory of the gravity model in migration studies. It turns out that the ideas embedded in the gravity model regarding economic and social processes date back to the 1880s. The ideas of the gravitational law:

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2} \quad (1)$$

are represented not only in one of the laws of migration but also in the mathematical description of global trade flows. From the point of view of physics, in (1)  $F$  is the attractive force, and  $M$  are the masses (mass  $i$  and mass  $j$ ),  $D$  is the distance between the centres of the two objects,  $G$  is a universal gravitational constant.

In general, for the laws of gravitation Soriano-Paños et al. (2022) has found wide application in the social sciences and describes the degree of interaction between *population centers*. In particular, it is noted there that the population flow  $T_{ij}$  between two locations is directly proportional to the product of their populations ( $N_i$  and  $N_j$ ) and inversely proportional to the distance ( $d_{ij}$ ) among them:

$$T_{ij} = K \frac{N_i N_j}{d_{ij}^2}. \quad (2)$$

In the paper by Kluge (2023), the research concerns mathematical questions of steady states in gravitational and relation models, as well as the study of the temporal dynamics under different scenarios. In particular, it has been theoretically established that the gravitational model has two long-term solutions, which depend on the ratio of its parameters and do not depend on the initial and spatial distributions of the population.

With the development of machine learning, its methods have begun to be applied in the study of migration flows. This has led to the emergence of the so-called hybrid approach to modeling, in which the classical gravitational theory is combined with machine learning methods. Many studies have been devoted to this research direction (e.g. Gu et al., 2024; Liu et al., 2024; Simini et al., 2021; Litmeyer et al., 2024; Ruzicka et al., 2024).

Of particular interest are practices of adapting the theory of human mobility within a single city (Kwon et al., 2023). This provides local authorities with tools for informed decision-making, infrastructure development, and urban transport planning. In the article by Bogusz et al. (2024), the problem of uneven migration flows from Warsaw to its suburban municipalities is studied. It was found that the determinants of such migration flows are the economic condition of the municipality, its level of urbanization, the availability of amenities, and the commuting time to the center of Warsaw.

The study by Zhang et al. (2024) draws attention to the weaknesses of the classical gravitational theory when describing human flows within a city. In particular, the population size used in the gravitational model as the main *attractive* force of migration flow and the physical distance within a city require refinement. The authors propose to use the concept of *activity space* as a basis and to consider the *urban region's attractiveness* as the *attractive force*. These refinements and innovations resulted in the so-called *activity space-based gravity (ASG) model*. The idea of *attractiveness* is also used as a basis for modelling intercity mobility in the study by Yu et al. (2024).

In the study by Wang et al. (2024), the migration model is considered from the perspective of travel. The authors developed a theory and constructed a model based on the assumption: "Here, we present a cost opportunity model that assumes an individual choosing a destination is proportional to the number of opportunities at the destination, and inversely to the power of the number of intervening opportunities between the origin and destination." A similar model, based on the relationship between opportunities at the origin and the destination, is described by Liu & Yan (2020).

Overall, the theory of human flows has also extended to other subject areas. For example, in the study by Łoś et al. (2023), animal migration is studied using an advanced mathematical framework of human mobility modeling.

The development of the concept of *mobility* is not limited to physical space. Such a space can also be, e.g., the space of scientific fields. This is confirmed by the research finding of Singh et al. (2024). In this work, the authors, analyzing the publication history of individual researchers, identified patterns of scientific mobility that strongly resemble physical mobility. Taken together, these trajectories form migration flows that can be described by a gravity model. In our opinion, scientific fields can be grouped into clusters between which migration operates according to the principle of rapid migration flows and transitions, and within which the principle of diffusion acts between adjacent, closely related fields. That is, when studying

migration processes, it is in fact insufficient to limit oneself to spatiotemporal distribution. It is necessary to transition to a social space, which also includes spatiotemporal characteristics. This requires the definitions of metrics in such social spaces. For example, studies of information spread in social networks carried out by Wang et al. (2012); Zhou et al. (2021); Foroozani & Ebrahimi (2021); Ganai et al. (2023) have faced the need to introduce *distance* metrics in social networks.

In the meantime, in the study by Capoani (2023) aspects of the *death of the gravity model* are discussed which are associated with the *death of distance*. However, we emphasize again that, in our opinion, the concept of *distance* here should be considered in a broader sense than just physical distance – namely, the transition into social space and the consideration of the social distance (Boguñá et al., 2004). The transition into social space and the consideration of special distance metrics (distance through friends, distance through interests) have been examined and interpreted in mathematical models of information spread in social networks (Wang et al., 2012; Dai et al., 2015; Zhou et al., 2021; Foroozani & Ebrahimi, 2021; Ganai et al., 2023).

We propose to consider going beyond the gravity model from another perspective. In the gravitational law, the factors of *attraction* are determined by the capacities of centers (either by population size or by production volumes). However, in the modern world, the factors that may drive the migration process are in fact *determinants*, which, depending on circumstances, include various economic and non-economic factors of attraction. Researchers most often include among them the labor market situation, income levels, and social programs in host countries (Berde & Remsei, 2025; Kochaniak et al., 2024; Ngwu et al., 2023; Yurchyk et al., 2023; Zavisca et al., 2023), overall well-being and satisfaction with life (Aliyev & Gasimov, 2023), as well as their partial manifestations in social relations, such as trust, fairness, the quantity of social services, and the observance of rights in various spheres (Chugaievska & Wisła, 2023; Mishchuk & Grishnova, 2015; Perelli-Harris et al., 2023). And although in the laws of the form of (1) and (2) these factors may enter into the functions  $G$  and  $K$ , we propose to treat these determinants as the ones forming the *system-shaping foundation* of migration flows.

### 3. Results

#### 3.1. Proposals on the Semi-Discrete Migration Flow

As is well known, the main driving forces of migration flows are specific determinants of these migration flows (Berde & Remsei, 2025; Chugaievska & Wisła, 2023; Kochaniak et al., 2024; Mishchuk & Grishnova, 2015; Ngwu et al., 2023; Perelli-Harris et al., 2023; Yurchyk et al., 2023; Zavisca et al., 2023). That is, if the determinant of a migration flow is the *level of wages*, then the potential function that may motivate a person to migrate – either geographically (physical space) or in terms of changing profession or workplace within the same city of residence (so-called migration in social space) – is precisely the *difference* in that determinant. The absence of a change in the determinant in some dimension of social or physical space implies the absence of any preconditions for a person to make a migration decision.

For now, let us consider the migration process in areas related to physical space. Later, by analogy, we will extend our reasoning to the case of social spaces as well. Let  $\Omega_x$  and  $\Omega_y$  be two bounded non-empty regions in the Euclidean space  $E_2$ . Moreover, let  $\Omega_x \cap \Omega_y = \emptyset$ ,  $X =$

$(x_1, x_2) \in \Omega_x$ ,  $Y = (y_1, y_2) \in \Omega_y$ . We will study the migration flow from the region  $\Omega_x$ . We define the magnitude of the migration flow intensity, as a vector function, in the following way:

$$u(X, Y, t) = k \frac{(F(X, t) - F(Y, t))^+}{r(X, Y)}. \quad (3)$$

Here,  $F(\cdot) = \{f_i(\cdot)\}_{i=1}^n$  is a vector function of  $n$  migration flow determinants;  $r(X, Y)$  is a function of the *distance* between two points  $X = (x_1, x_2) \in \Omega_x$ ,  $Y = (y_1, y_2) \in \Omega_y$  in two different regions;  $(\cdot)^+$  is the positive part of a function, defined as  $(v)^+ = \begin{cases} v, & v \geq 0; \\ 0, & v < 0; \end{cases}$   $k$  is a proportionality coefficient;  $u(\cdot) = \{u_i(\cdot)\}_{i=1}^n$  is the migration flow vector. According to equation (3), if  $(f_i(X, t) - f_i(Y, t)) > 0$ , then this may cause a migration flow from the point  $Y = (y_1, y_2) \in \Omega_y$  to the point  $X = (x_1, x_2) \in \Omega_x$ . It should be noted that in principle  $F(X, t)$  and  $F(Y, t)$  are different vector functions, and only to reduce the number of symbols we use the same letter  $F$  for both; we will distinguish them by their argument.

Our idea behind moving to equation (3), namely using the *gradient* of determinants over the distance  $r$ , is conceptually based on Darcy's Law for fluid flow in a saturated porous environment (Whitaker, 1986). The determinant of fluid movement in a saturated porous environment is the gradient of hydraulic head along the path of filtration. In our case, the hydraulic head acts as the potential of the pore fluid flow, and analogously, the determinants' functions over respective distances in social space represent the potentials for migration flows.

Furthermore, by analogy with generalized forms of Darcy's Law, in equation (3), the function  $k$  may take a tensor form, i.e.,  $k = \{k_{ij}\}_{i=1, j=1}^{n, n}$ . This means that the migration flow component  $u_i$ , which corresponds to the determinant  $f_i$ , can actually be influenced not only by changes in this particular determinant but also by changes in other determinants

$$u_i = \sum_{j=1}^m k_{ij} \frac{(f_j(X, t) - f_j(Y, t))^+}{r(X, Y)}, i = \overline{1, n}.$$

### 3.2. Migration Equations

According to our proposed equation (3), the possible intensity of the migration flow is defined. Let us consider as unknowns the functions that represent the number of people at a given time  $t$  located in the regions  $\Omega_y$  and  $\Omega_x$ . We denote them as  $n(Y, t)$  and  $N(X, t)$ , respectively. Then, based on the ideas of information dissemination in social networks, we arrive at the following system of migration equations:

$$\frac{\partial N(X, t)}{\partial t} = \nabla \cdot (D_x \nabla N) + un(Y, t) \left(1 - \frac{n(Y, t)}{n_{max}}\right) - vN(X, t) \left(1 - \frac{N(X, t)}{N_{max}}\right), X = (x_1, x_2) \in \Omega_x, Y = (y_1, y_2) \in \Omega_y; \quad (4)$$

$$\frac{\partial n(Y, t)}{\partial t} = \nabla \cdot (D_Y \nabla n) - un(Y, t) \left(1 - \frac{n(Y, t)}{n_{max}}\right) + vN(X, t) \left(1 - \frac{N(X, t)}{N_{max}}\right), X = (x_1, x_2) \in \Omega_x, Y = (y_1, y_2) \in \Omega_y. \quad (5)$$

Here,  $n_{max}$ ,  $N_{max}$  represent the maximum number of people in the regions  $\Omega_y$  and  $\Omega_x$  respectively. The first terms on the right-hand sides of equations (4) and (5) are diffusion terms of the migration flows within the regions  $\Omega_y$  and  $\Omega_x$  and represent internal processes. The

remaining terms represent external migration flows, which are determined by the potential functions of the determinants. In particular,

$$v(Y, X, t) = K \frac{(F(Y, t) - F(X, t))^+}{r(X, Y)}$$

and, in general case, the determinant components of the functions  $F(X, t)$  and  $F(Y, t)$  may differ from those used in equation (3).

#### 4. Discussion and Interpretation of the Results

The gravitational theory may perform well in terms of predicting processes based on given data. However, from the perspective of cybernetic system – such as social system and society as a whole – the issue of process control based on decision theory plays an important role in the construction of mathematical models. Therefore, in our view, migration models based on flows, from this standpoint, deserve further development as an extension beyond gravitational theory.

Systems (4) and (5) constitute a system of partial differential equations (PDEs). The description of migration flows using PDEs has its advantages and disadvantages. The main advantage is the possibility of applying the already developed apparatus of mathematical physics equations, including numerical and approximate methods. In particular, we believe it is worth mentioning the theory of PDEs on graphs (Van Gennip & Budd, 2025). The advanced control theory in PDEs offers hope for the development of this direction in relation to managing migration flows and making corresponding decisions at the state and regional levels. Another disadvantage is the potential for further **generalization of migration models based on the equations of mathematical physics**:

1. It can be easily generalized to account for nonlinear effect in migration models. This includes the presence of a *threshold gradient* and the incorporation of nonlinear effects in the law governing migration flows. A threshold gradient refers to the initiation of a migration flow only when the change in a determinant exceeds a certain value.
2. The application of migration flows in models of information dissemination and epidemic spread is also based on PDEs.
3. The presence of *osmotic* effects through external control functions enables the development of tools for managing migration flows. Again, from the standpoint of rigor, this theory is well developed in cybernetic models based on PDEs.
4. The application of the concept of *diffusion* at regional and small-scale levels of population migration studies, which are poorly described by the gravity law. The presence of a convective part in the diffusion coefficient allows the use of determinant gradients at small scales.
5. Any other determinant of the flow can be easily introduced, which may become an important factor in restraining population or labor force outflows (e.g., improving social standards or the environmental attractiveness of a region as a substitute for insufficient wage levels).
6. The possibility of applying *filtering* models, such as the Mincer model, by analogy with porous media when considering the social environment. This applies, e.g., to restraining the outflow of clearly defined professional or age group.

7. The development of probabilistic models within the PDE framework offers hope for their application to migration flow models based on PDEs.
8. The ability to take into account borders between regions and countries as *thin geobarriers* with their own *semi-permeable* and *osmotic* properties, which affect migration flows. By the way, such models can also be used to describe flows between sectors of the economy to stimulate or reduce labor outflow and transition from one sector to another.
9. The model can be easily extended to different age groups using clustering methods and making certain assumptions about transitions between groups. As a result, we obtain systems of interrelated differential equations.
10. Further developing idea (9) requires introducing the concept of *social distance* and considering PDEs in so-called *social space*. From there, everything depends on its dimensionality. However, this demonstrates the possibility of applying PDEs in spaces with dimensions greater than *three*.
11. Advanced methods of calibrating mathematical models and assimilating external observation data – more broadly, Data Fusion methods – can also be applied to problems involving equations of types (4), (5) within the PDE framework. Data assimilation allows for real-time forecast correction and modification of control functions in accordance with current data.

## 5. Conclusion

The article draws attention to the relevance in today's world of the phenomenon of *human mobility* in a broad sense – from emigration and tourism to the system of commuter transportation between *workplaces and places of residence* in large metropolitan areas. From the standpoint of mathematical formalization of these processes and the formulation of mathematical-computer models, in our view, attention should first and foremost be paid to the purpose of such formalizations. The goal of constructing mathematical models of human mobility is to develop *optimal* tools for the effective management of these processes, to support decision-making in the socio-economic domains of communities, cities, and states. By *optimal* we mean tools that are sufficiently accurate while remaining cost-effective. Therefore, in this article, we propose to focus on mathematical models described by boundary value problems of mathematical physics, especially on *diffusion-convective* type models. To this end, we suggest generalizing the gravity model for human flows. Specifically, we propose considering the flow of individuals from the perspective of a potential function of determinants and its gradients in a social space. This is analogous to potential fluid flow in a porous medium, where the potential function is the hydraulic head (pressure). The proposed approach assumes a graph-continuous structure of the research domain. In particular, the vertices of the graph represent continuous subsets of social space, where mobility processes are described using the diffusion principle. At the same time, mobility processes along the graph's edges, between the continuous regions (vertices), are described by convective flows based on gradients of determinant functions. The approach proposed by the authors has significant potential for further development and practical implication. Further studies can be based both on the relatively new theory of PDEs on graphs, and on the substantial achievements in classical boundary value problem theory, numerical methods, and control theory of analogous physical and chemical processes.



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